Quantum Homework Template

Erin De Pree

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Suppressing Section Numbering

If you are going to label your sections by problem number, then it is a good idea to use `\section*` instead of `\section` so you can suppress the section numbering. This way you won't write "3 Problem 2"!

Here, I'm going to work through problem 1.5 from our text. This will show you how to write kets and such. I've also included a section on matrices so you can see how to write them up.

Be sure to include the `depreequantum.sty` style file as it loads the necessary packages.

Problem 1.5

A beam of spin-1/2 particles is prepared in the state

$$|\psi\rangle = \tfrac{2}{\sqrt{13}}\,|+\rangle + i\tfrac{3}{\sqrt{13}}\,|-\rangle$$

Part a What are the possible results of a measurement of the spin component S_z, and with probabilities would they occur?

You will either measure $S_z = +\hbar/2$ (spin up) or $S_z = -\hbar/2$ (spin down).

The probability of measuring $S_z = +\hbar/2$ is

$$\begin{aligned}\mathcal{P}_+ = |\langle +|\psi\rangle|^2 &= \left|\langle +|\left(\frac{2}{\sqrt{13}}\,|+\rangle + i\frac{3}{\sqrt{13}}\,|-\rangle\right)\right|^2 \\ &= \left|\frac{2}{\sqrt{13}}\,\langle +|+\rangle + i\frac{3}{\sqrt{13}}\,\langle +|-\rangle\right|^2 \\ &= \left|\frac{2}{\sqrt{13}} + 0\right|^2 = \boxed{\frac{4}{13} \simeq 31\%}\end{aligned}$$

While the probability of measuring $S_z = -\hbar/2$ is

$$\begin{aligned}
\mathcal{P}_- = |\langle -|\psi\rangle|^2 &= \left|\langle -| \left(\frac{2}{\sqrt{13}} |+\rangle + i\frac{3}{\sqrt{13}} |-\rangle \right)\right|^2 \\
&= \left| \frac{2}{\sqrt{13}} \langle -|+\rangle + i\frac{3}{\sqrt{13}} \langle -|-\rangle \right|^2 \\
&= \left| 0 + i\frac{3}{\sqrt{13}} \right|^2 = \left(-i\frac{3}{\sqrt{13}}\right)\left(i\frac{3}{\sqrt{13}}\right) \\
&= -i^2\frac{9}{13} = \boxed{\frac{9}{13} \simeq 69\%}
\end{aligned}$$

Part b What are the possible results of a measurement of the spin component S_x, and with probabilities would they occur?

Measure $S_x = \pm\hbar/2$.

Probability of measuring $S_x = +\hbar/2$ is

$$\begin{aligned}
\mathcal{P}_{+x} = |_x \langle +|\psi\rangle|^2 &= \left| \left(\frac{1}{\sqrt{2}} \langle +| + \frac{1}{\sqrt{2}} \langle -| \right)\left(\frac{2}{\sqrt{13}} |+\rangle + i\frac{3}{\sqrt{13}} |-\rangle \right) \right|^2 \\
&= \left| \frac{1}{\sqrt{2}}\frac{2}{\sqrt{13}} + i\frac{1}{\sqrt{2}}\frac{3}{\sqrt{13}} \right|^2 = \left| \frac{2+3i}{\sqrt{26}} \right|^2 \\
&= \frac{1}{26}(2-3i)(2+3i) = \frac{4-6i+6i-9i^2}{26} \\
&= \frac{4+9}{26} = \frac{13}{26} = \boxed{\frac{1}{2} = 50\%}
\end{aligned}$$

Probability of measuring $S_x = -\hbar/2$ is

$$\begin{aligned}
\mathcal{P}_{-x} = |_x \langle -|\psi\rangle|^2 &= \left| \left(\frac{1}{\sqrt{2}} \langle +| - \frac{1}{\sqrt{2}} \langle -| \right)\left(\frac{2}{\sqrt{13}} |+\rangle + i\frac{3}{\sqrt{13}} |-\rangle \right) \right|^2 \\
&= \left| \frac{1}{\sqrt{2}}\frac{2}{\sqrt{13}} - i\frac{1}{\sqrt{2}}\frac{3}{\sqrt{13}} \right|^2 = \left| \frac{2-3i}{\sqrt{26}} \right|^2 \\
&= \frac{1}{26}(2+3i)(2-3i) = \frac{4-6i+6i-9i^2}{26} \\
&= \frac{4+9}{26} = \frac{13}{26} = \boxed{\frac{1}{2} = 50\%}
\end{aligned}$$

Part c Histograms! See fig. 1 for the histograms. Also note the figure appears on the next page because LaTeX is automatically placing it where it makes most sense (after it is referenced).

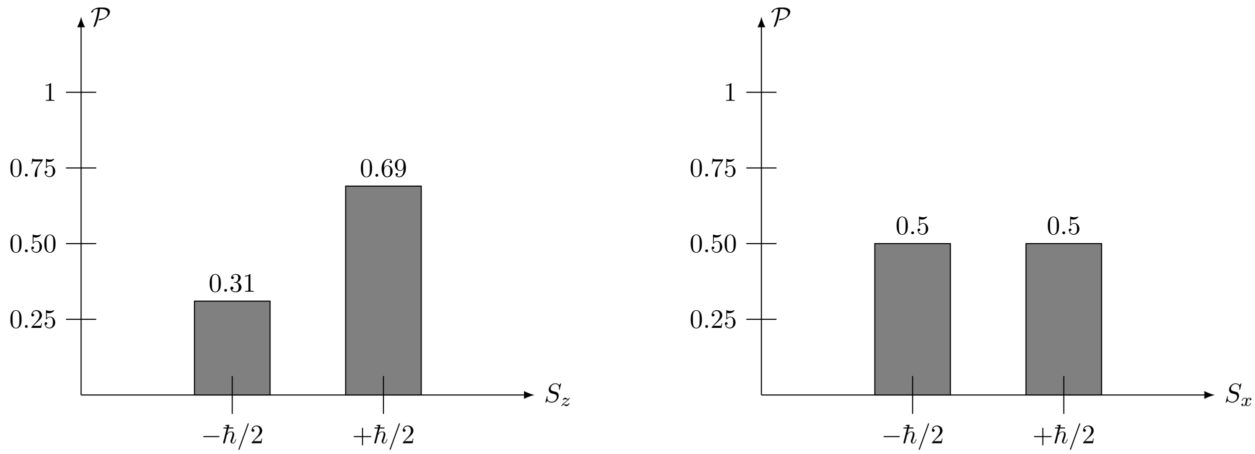

Figure 1: Probabilities of measuring various spins in problem 1.5. You may find it easier to make your histograms in Excel or Google Sheets or by hand and include them as a picture.

Matrix Notation

Going to use $\doteq$ to mean *represented by*. In LaTeX, use the notation: `\dot{=}`. The quantum state isn't a column vector, but it can be represented by a column vector

$$|+\rangle \doteq \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |-\rangle \doteq \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{1}$$

This is the z-basis set.

Let's use the column vector notation to represent a state $|\psi\rangle$ as a projection of the state on the $|+\rangle$ and the projection on the $|-\rangle$:

$$|\psi\rangle \doteq \begin{pmatrix} \langle +|\psi\rangle \\ \langle -|\psi\rangle \end{pmatrix}$$

$$|\psi\rangle = a\,|+\rangle + b\,|-\rangle$$

$$|\psi\rangle \doteq \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\langle\psi| \doteq \begin{pmatrix} a^* & b^* \end{pmatrix}$$

We can also write the x and y kets in the z-basis:

$$|+\rangle_x \doteq \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad |+\rangle_y \doteq \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$|-\rangle_x \doteq \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad |-\rangle_y \doteq \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$$