

### Abstract

No modelo que a Danielle postou não tinha resumo, então confirmem para saber se precisa ou não. Morphological pattern spectra computed from granulometries are frequently used to classify the size classes of details in textures and images. An extension of this technique, which retains information on the spatial distribution of the details in each size class is developed. Algorithms for computation of these spatial pattern spectra for a large number of granulometries on binary images are presented.

### 1. Introdução

A importância do assunto deve ser destacada resumidamente.

GRANULOMETRIES are ordered sets of morphological openings or closings, each of which removes image details below a certain size. These can be used for texture analysis through the use of *pattern spectra*, which show how the number of foreground pixels in the image changes as a function of the size parameter [3]. A drawback of the classical definition of pattern spectra is that spatial information is not included in a pattern spectrum as shown below. In this paper, *spatial pattern spectra* are developed which retain information on the distribution of these details at different scales.

**Figure 1:** Parts (a) through (c) show three images consisting of squares of different sizes; (d) shows the pattern spectra, denoting the number of foreground pixels removed by openings by reconstruction by  $\lambda \times \lambda$  squares. No granulometry is capable of separating the patterns, because the only differences between the images lie in the distributions of the connected components.

### 2. Objetivos

Dar uma ideia compacta da metodologia ou forma de abordagem da pesquisa, bem como o projeto foi desenvolvido.

Let binary images  $X$  and  $Y$  be defined as a subset of the image domain  $M \subset \mathbb{Z}^n$  or  $\mathbb{R}^n$  (usually  $n = 2$ ).

**Definition 1** A binary granulometry is a set of operators  $\{\alpha_r\}$  with  $r$  from some ordered set  $\Lambda$  (usually  $\Lambda \subset \mathbb{R}$  or  $\mathbb{Z}$ ), with the following three properties

$$\begin{aligned} \alpha_r(X) &\subset X & (1) \\ X \subset Y &\Rightarrow \alpha_r(X) \subset \alpha_r(Y) & (2) \\ \alpha_r(\alpha_s(X)) &= \alpha_{\max(r,s)}(X), & (3) \end{aligned}$$

for all  $r, s \in \Lambda$ .

**Definition 2** The pattern spectrum  $s_\alpha(X)$  obtained by applying granulometry  $\{\alpha_r\}$  to a binary image  $X$  is defined as

$$(s_\alpha(X))(u) = -\left. \frac{\partial A(\alpha_r(X))}{\partial r} \right|_{r=u} \quad (4)$$

in which  $A(X)$  is a function denoting the Lebesgue measure in  $\mathbb{R}^n$ .

In the case of discrete images, and with  $r \in \Lambda \subset \mathbb{Z}$ , this differentiation reduces to

$$(s_\alpha(X))(r) = \#(\alpha_r(X) \setminus \alpha_{r+}(X)) \quad (5)$$

$$= \#(\alpha_r(X)) - \#(\alpha_{r+}(X)), \quad (6)$$

with  $r^+ = \min\{r' \in \Lambda | r' > r\}$ , and  $\#(X)$  the number of elements of  $X$ .

The opening transform [5]  $\Omega_X$  of a binary image  $X$  for a granulometry  $\alpha_r$  is

$$\Omega_X(x) = \max\{r \in \Lambda | x \in \alpha_r(X)\} \quad (7)$$

The pattern spectrum of a binary image  $X$  using granulometry  $\{\alpha_r\}$  is the histogram of  $\Omega_X$  obtained with the same size distribution [5], disregarding the bin for grey level 0.

**Figure 2:** Opening transform with  $\{\alpha_r\}$  as in Fig. 1: (left) original image; (right) opening transform (contrast stretched for clarity).

### 3. Metodologia

Dar uma ideia compacta da metodologia ou forma de abordagem da pesquisa, bem como o projeto foi desenvolvido. Pattern spectra only retain the amount of detail present at scale  $r$ . This can be amended by computing some parameterization of the spatial distribution in an image  $\alpha_r(X) \setminus \alpha_{r+}(X)$  as a function of  $r$ .

**Definition 3** Let  $M(X)$  be some parameterization of the spatial distribution of detail in the image  $X$ . The spatial pattern spectrum  $S_{M,\alpha}$  is then defined as

$$(S_{M,\alpha}(X))(r) = M(\alpha_r(X) \setminus \alpha_{r+}(X)). \quad (8)$$

An obvious parameterization of the spatial distribution is through the use of moments. Focusing on the case of 2-D binary images, the moment  $m_{ij}$  of order  $ij$  of an image  $X$  is given by

$$m_{ij}(X) = \sum_{(x,y) \in X} x^i y^j. \quad (9)$$

The spatial moment spectrum  $S_{m_{ij},\alpha}$  of order  $ij$  is

$$(S_{m_{ij},\alpha}(X))(r) = m_{i,j}(\alpha_r(X) \setminus \alpha_{r+}(X)). \quad (10)$$

For  $i = 0$  and  $j = 0$  we obtain the standard pattern spectrum. For each  $r$ ,  $(S_{m_{0,0},\alpha}(X))(r)$  is just the moment of an image, therefore, derived parameters such as coordinates of the centre of mass, (co-)variances, skewness and kurtosis of the distribution of details at each scale can be computed easily. We can then define pattern mean spectra, pattern (co-)variance spectra, pattern kurtosis spectra, etc. The pattern mean- $x$  and variance- $x$  spectra ( $S_{\bar{x},\alpha}$  and  $S_{\sigma(x),\alpha}$ ) are defined as:

$$S_{\bar{x},\alpha} = \frac{S_{m_{1,0},\alpha}}{S_{m_{0,0},\alpha}} \quad (11)$$

and

$$S_{\sigma(x),\alpha} = \sqrt{\frac{S_{m_{2,0},\alpha}}{S_{m_{0,0},\alpha}} - S_{\bar{x},\alpha}^2}. \quad (12)$$

These two are shown in Figures 3 and 4. Note that these definitions hold only where  $(S_{m_{0,0},\alpha}(f))(r) \neq 0$ . For all other values of  $r$  they will be defined as zero. Further post-processing can be done to compute central moments and moment invariant from pattern moment spectra [1, 2].

### 4. Resultados e Discussão

Verificar os principais resultados obtidos de acordo com os objetivos propostos.

Nacken [5] derived an algorithm for computation of pattern spectra for granulometries based on openings by discs of increasing radius for various metrics, using the opening transform. After the opening transform has been computed, it is straightforward to compute the pattern spectrum:

- Set all elements of array  $s$  to zero
- For all  $x \in X$  increment  $s[\Omega_X(x)]$  by one.

To compute the pattern *moment* spectrum, the only thing that needs to be changed is the way  $s[\Omega_X(x)]$  is incremented. As shown in Algorithm 1.

- Set all elements of array  $s$  to zero
- For all  $(x, y) \in X$  increment  $s[\Omega_X(x, y)]$  by  $x^i y^j$ .

**Algorithm 1:** Algorithm for computation of pattern moment spectrum of order  $ij$ .

This algorithm can readily be adapted to other granulometries, simply by computing the appropriate opening transform.

**Figure 3:** The opening transform using city-block metric: (a) opening transform of Fig. 1(c); (b) pattern spectrum; (c) pattern variance- $x$ ; (d) variance- $y$  spectra.

**Figure 4:** Pattern mean- $x$  (top) and variance- $x$  (bottom) spectra: the three columns show spectra for Fig. 1(a), (b) and (c) from left to right respectively. Unlike the standard pattern spectra, these spatial pattern spectra can distinguish the three images.

### 5. Conclusão

Sitting on a corner all alone, staring from the bottom of his soul, watching the night come in from the window  
It'll all collapse tonight, the fullmoon is here again  
In sickness and in health, understanding so demanding  
It has no name, there's one for every season  
Makes him insane to know

Running away from it all "I'll be safe in the cornfields", he thinks  
Hunted by his own, again he feels the moon rising on the sky

Find a barn which to sleep in, but can he hide anymore  
Someone's at the door, understanding too demanding  
Can this be wrong, it's love that is not ending  
Makes him insane to know

She should not lock the open door (Run away, run away, run away)  
Fullmoon is on the sky and He's not a man anymore  
sees the change in him but can't (Run away, run away, run away)  
See what became out of her man Fullmoon  
Swimming across the bay, the night is gray, so calm today  
She doesn't wanna wait. "We've gotta make the love complete tonight..."

In the mist of the morning he cannot fight anymore  
Hundred moons or more, he's been howling  
Knock on the door, and scream that is soon ending  
Mess on the floor again

She should not lock the open door (Run away, run away, run away)  
Fullmoon is on the sky and he's not a man anymore  
She sees the changes in him but can't (Run away, run away, run away)  
See what became out of her man

She should not lock the open door (Run away, run away, run away)  
Fullmoon is on the sky and he's not a man anymore  
sees the changes in him but can't (Run away, run away, run away)  
See what became out of her darling man

She should not lock the open door (Run away, run away, run away)  
Fullmoon is on the sky and he's not a man anymore  
See what became out of her man

### References

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