ELEC 340 — Applied Electromagnetics and Photonics

Name: David Li **Due Date:** January 25 2018, 11:30 AM
Student Number: V00818631 **Assignment:** Number 3

Problem 1 Given that the electric field in free space is:

$$E(R,\theta,t) = \hat{\theta}\frac{2}{R}\sin(\theta)\cos(6\pi\times 10^9 t - 2\pi R)\ \text{mV/m}$$

where R and θ are the radial and polar variable in the spherical coordinate system. Find:

(a) The phasor representation of the given electric field vector.
$\tilde{\mathbf{E}}(R,\theta) = \hat{\theta}E_\theta = \hat{\theta}\frac{2}{R}\sin(\theta)e^{-j2\pi R}\text{mV/m}$

(b) The phasor representation of the associated magnetic field vector.

$$\nabla\times\tilde{\mathbf{E}} = j\omega\mu\tilde{\mathbf{H}}$$
$$\tilde{\mathbf{H}} = \frac{1}{j\omega\mu}\nabla\times\tilde{\mathbf{E}}$$

$$\nabla\times\tilde{\mathbf{E}} = \frac{1}{R\sin\theta}\left(\frac{\partial}{\partial\theta}(E_\phi sin(\theta)) - \frac{\partial E_\theta}{\partial\phi}\right)\hat{\mathbf{R}} + \frac{1}{R}\left(\frac{1}{\sin(\theta)}\frac{\partial E_R}{\partial\phi} - \frac{\partial}{\partial R}(RE_\phi)\right)\hat{\theta} + \frac{1}{R}\left(\frac{\partial}{\partial R}(RE_\theta) - \frac{\partial E_R}{\partial\theta}\right)\hat{\phi}$$

Since $\tilde{\mathbf{E}}$ only has non-zero values in the $\hat{\phi}$ direction.

$$\nabla\times\tilde{\mathbf{E}} = \frac{1}{r\sin\theta}\left(-\frac{\partial E_\theta}{\partial\phi}\right)\hat{\mathbf{R}} - \frac{1}{R}\frac{\partial}{\partial R}(RE_\theta)\hat{\phi} = -\left(\frac{1}{R\sin\theta}\left(\frac{\partial E_\theta}{\partial\phi}\right)\hat{\mathbf{R}} + \frac{1}{R}\frac{\partial}{\partial R}(RE_\theta)\hat{\phi}\right)$$

$$\begin{aligned}
\tilde{\mathbf{H}} &= -\frac{1}{j\omega\mu}\nabla\times\tilde{\mathbf{E}} = \frac{-1}{j\omega\mu}\hat{\phi}\frac{0.002}{R}\sin\theta\frac{\partial}{\partial R}(e^{-j2\pi R})\\
&= \hat{\phi}\frac{2\pi}{j6\pi\times 10^9\times 4\pi\times 10^{-7}}\frac{0.002}{R}\sin(\theta)e^{-j2\pi R}\\
&= \hat{\phi}\frac{5.30516477\times 10^{-7}}{R}\sin(\theta)e^{-j2\pi R-\pi/2}\ \ (\text{A/m})\\
&= \hat{\phi}\ \frac{53}{R}sin(\theta)e^{-j2\pi R-\pi/2}\ \ (\mu\text{A/m})
\end{aligned}$$

(c) The time-domain representation of the magnetic field you obtained in (b).

$$= \hat{\phi}\ \frac{53}{R}\sin(\theta)\cos(6\pi\times 10^9 t - 2\pi R - \pi/2)\ (\mu\text{A/m})$$

Problem 2 The electric field intensity of a 5-MHz linearly polarized uniform plane wave traveling in free space in 10 V/m. The electric field is polarized in the +z direction at t = 0 and the wave in propagating in the -y direction. Find:

(a) The angular frequency and wave number, and intrinsic wave impedance.
Since the plane wave is traveling in free space, $\mu = \mu_0$ and $\epsilon = \epsilon_0$.

$\omega = 2\pi f = 10\pi \text{MHz} \quad k = \omega\sqrt{\mu\epsilon} = 10\pi \ \times 10^6 \times 3.335641 \times 10^{-9} = 0.0334\pi \text{ rad/m}$

The intrinsic impedance of a lossless medium is defined as:

$$\eta = \frac{\omega\mu}{k} = \frac{\omega\mu}{\omega\sqrt{\mu\epsilon}} = \frac{\mu}{\epsilon} \qquad (\Omega) \tag{1}$$

Since free space is being used: $\eta = \eta_0 = 120\pi\Omega$

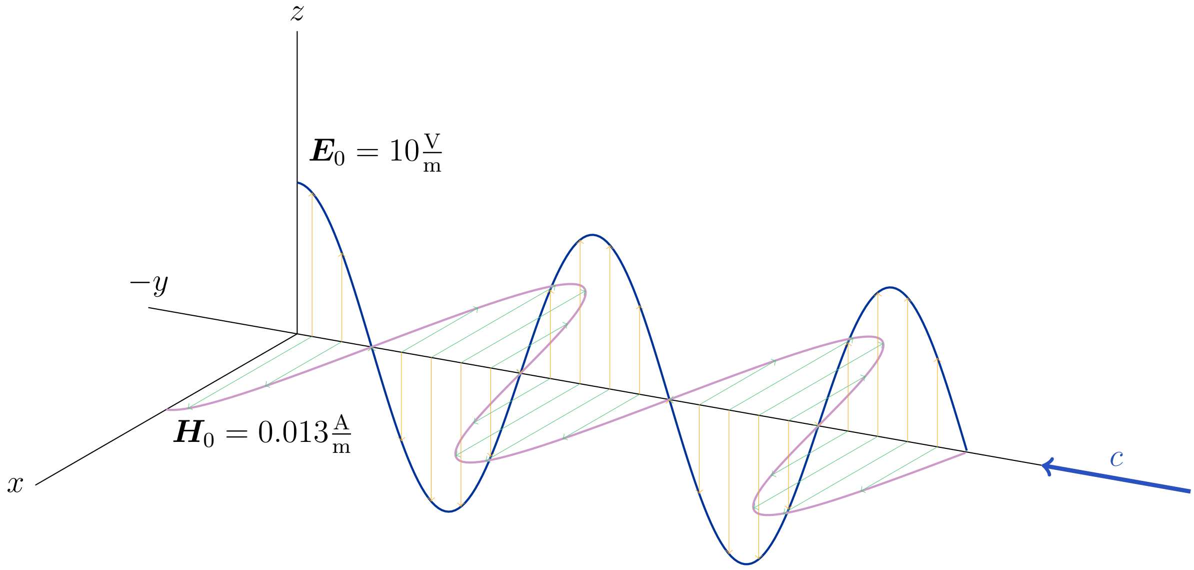
(b) The field vectors phasor, i.e. $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{H}}$

$$\begin{aligned}
\tilde{\mathbf{E}} &= -\hat{\mathbf{y}}E_0 = -\hat{\mathbf{y}}10e^{-j0.0334\pi z}\text{V/m} \\
\nabla \times \tilde{\mathbf{E}} &= \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\hat{\mathbf{z}} \\
\tilde{\mathbf{H}} &= \frac{1}{j\omega\mu}\nabla \times \tilde{\mathbf{E}} = -\frac{1}{j\omega\mu}\frac{\partial E_y}{\partial z}\hat{\mathbf{x}} = \frac{10(0.0334\pi)e^{-j0.0334\pi z}\hat{\mathbf{x}}}{j \times 2\pi \times 10 \times 10^6 \times 4\pi \times 10^{-7}} \\
&= \hat{\mathbf{x}}\ \frac{0.08339}{2\pi}e^{-j0.0334\pi z - \pi/2} \text{ A/m}
\end{aligned}$$

(c)

$$\begin{aligned}
\mathbf{E} &= -\hat{\mathbf{y}}10\cos(2\pi 10 \times 10^6 t - 0.0334\pi z) \text{ V/m} \\
\mathbf{H} &= \ \ \hat{\mathbf{x}}\ 0.0132719\cos(2\pi 10 \times 10^6 t - 0.0334\pi z - \pi/2) \text{ A/m}
\end{aligned}$$

(d) Draw a diagram to illustrate the field vectors and propagation direction.



$$\tilde{\mathbf{H}} = \frac{\nabla \times \tilde{\mathbf{E}}}{j\omega\mu}$$

E_0 = electric field amplitude
H_0 = magnetic field amplitude
c = speed of light (3×10^8m/s)

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

μ_0 = magnetic permeability in a vacuum, $\mu_0 = 1.3 \times 10^{-6}\,\text{N/A}^2$
ε_0 = electric permeability in a vacuum, $\varepsilon_0 = 8.9 \times 10^{-12}\,\text{C}^2/\text{Nm}^2$

Problem 3 Suppose that a uniform plane wave is traveling in the +x direction in a lossless dielectric ($\mu_r = 1$) with the 100 V/m electric field in the +z direction. If the wavelength is 25 cm and the velocity of propagation is 2×10^8m/s. Find:

(a) The relative permittivity ϵ_r and impedance η of the medium.

$$u_p = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\epsilon_r = \frac{1}{\mu_0 \mu_r \epsilon_0 u_p^2} = \frac{(1)}{4\pi \times 10^{-7}\text{H/m } 8.85 \times 10^{-12}\text{F/m } (2 \times 10^8 \text{ m/s})^2} = 2.24795$$

$$\eta = \frac{\mu}{\epsilon} = \frac{\mu_r \mu_0}{\epsilon_r \epsilon_0} = \frac{4\pi \times 10^{-7} \text{ H/m}}{2.24795 \times 8.85 \times 10^{-12} \text{ F/m}} = \frac{120\pi}{2.24795}\Omega = 53.38197\pi\ \Omega$$

(b) The angular frequency ω and the wave number k.

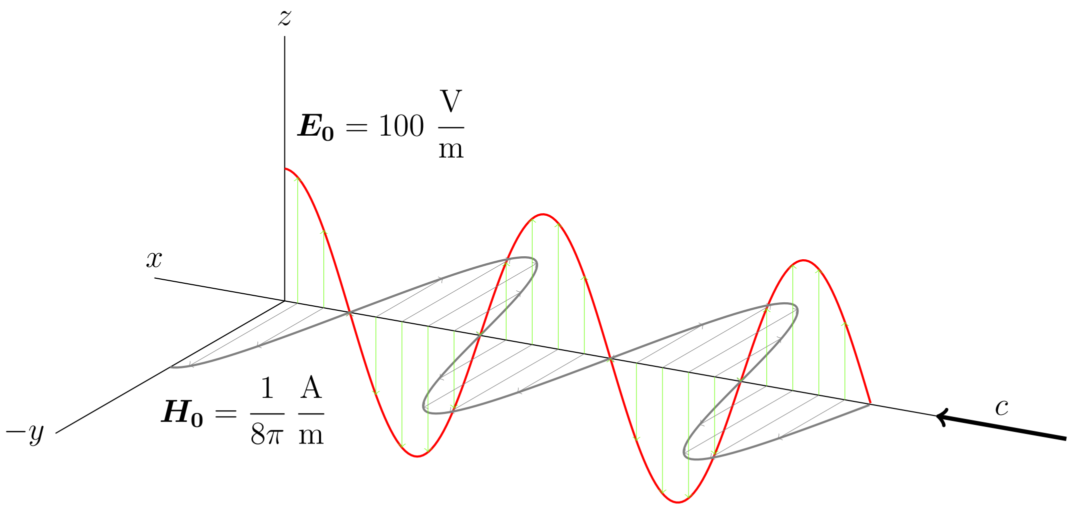
$$\text{Wave Number:} \quad k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.25 \text{ m}} = 8\pi \text{ rad/m}$$
$$\omega = \mu_p \times k = 2 \times 10^8 \text{ m/s} \times 8\pi \text{ rad/m} = 16\pi \times 10^8 \text{rad/s}$$

(c) The time-domain expressions for the electric and magnetic field vectors.

$$\tilde{\mathbf{E}} = \hat{\mathbf{x}}\tilde{E}_0 = \hat{\mathbf{x}}100e^{-jkz}\cos(\omega t - kz) = \hat{\mathbf{x}}100e^{-8\pi z} \text{ V/m}$$
$$\nabla \times \tilde{\mathbf{E}} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\hat{\mathbf{z}}$$
$$\tilde{\mathbf{H}} = \frac{1}{j\omega\mu}\nabla \times \tilde{\mathbf{E}} = \frac{1}{j\omega\mu}\frac{\partial E_x}{\partial z}\hat{\mathbf{y}} = \frac{100(-8\pi)e^{-j8\pi z}\hat{\mathbf{y}}}{j \times 16\pi \times 10 \times 10^8 \times 4\pi \times 10^{-7}} = -\hat{\mathbf{y}}\frac{1}{8\pi}e^{-j8\pi z - \pi/2}$$

$$\mathbf{E} = \hat{\mathbf{x}}E_0 = \hat{\mathbf{x}}100\cos(\omega t - kz) = \hat{\mathbf{x}}100\cos(16\pi \times 10^8 t - 8\pi z) \text{ V/m}$$
$$\mathbf{H} = -\hat{\mathbf{y}}\frac{1}{8\pi}\cos(16\pi \times 10^8 t - 8\pi z - \pi/2) \text{ A/m}$$

(d) Draw a diagram to illustrate the field vectors and propagation direction.



$$\tilde{\mathbf{H}} = \frac{\nabla \times \tilde{\mathbf{E}}}{j\omega\mu}$$

E_0 = electric field amplitude
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$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

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