2.2 Formal Write Up

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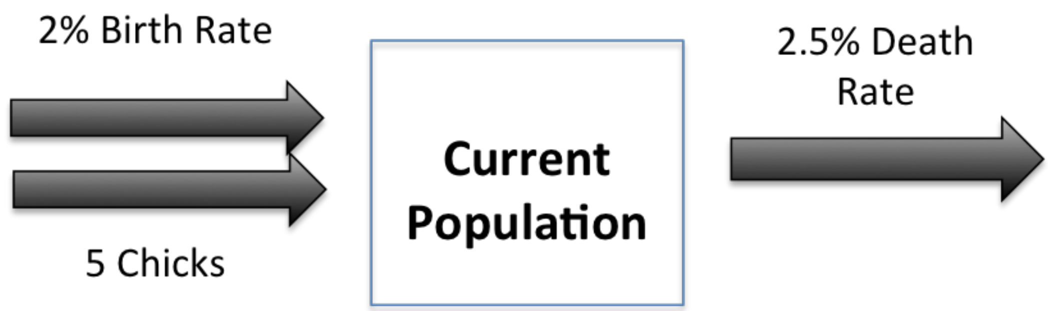
Problem 2.2.10

In this problem, you will create a model of a crane population that includes "hacking" chicks, which means to hatch chicks in captivity and release them into the wild. Assume that your crane population has a constant birth rate per year of 2% of the population and a constant death rate of 2.5%. Assume also that 5 chicks are "hacked" each year; i.e. assume 5 hatched chicks are released into the wild and absorbed into the population.

1. Draw a compartmental diagram for your model.



2. Use the compartmental diagram to create your model.

Looking at this compartmental diagram, we can see that the model for this problem will determine each year's population by having the birth rate added to the population, 5 hacked chicks added into the population, and the death rate taken out of the population. This model can be represented as such:

$$P(t+1) = 0.995P(t) + 5$$

with P representing the population and t representing time in years.

3.What are the fixed points of your model? Are they stable or unstable? How can you interpret this physically?

To find the fixed point of this model, we can find when $f(\bar{a}) = \bar{a}$.

$$P(t+1) = 0.995P(t) + 5$$

$$\bar{a} = 0.995\bar{a} + 5$$

$$0.005\bar{a} = 5$$

$$\bar{a} = 1000$$

To find out if this fixed point is stable or unstable, we need to find out if $|f'(\bar{a})|$ is less than or greater than 1.

$$P(t+1) = 0.995P(t) + 5$$

$$f(\bar{a}) = 0.995\bar{a} + 5$$

$$f'(\bar{a}) = 0.995$$

Since $f'(\bar{a})$ is less than one, this tells us that the fixed point is stable, or in other words, an attracting fixed point. If we were to interpret our fixed point physically, we would say that once the population reaches 1,000, it will no longer increase nor will it decrease because the birth and death rate will result in a population of 995 plus the 5 hacked chicks every year from there on out.