

The Sine Rule

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1 The Sine Rule:

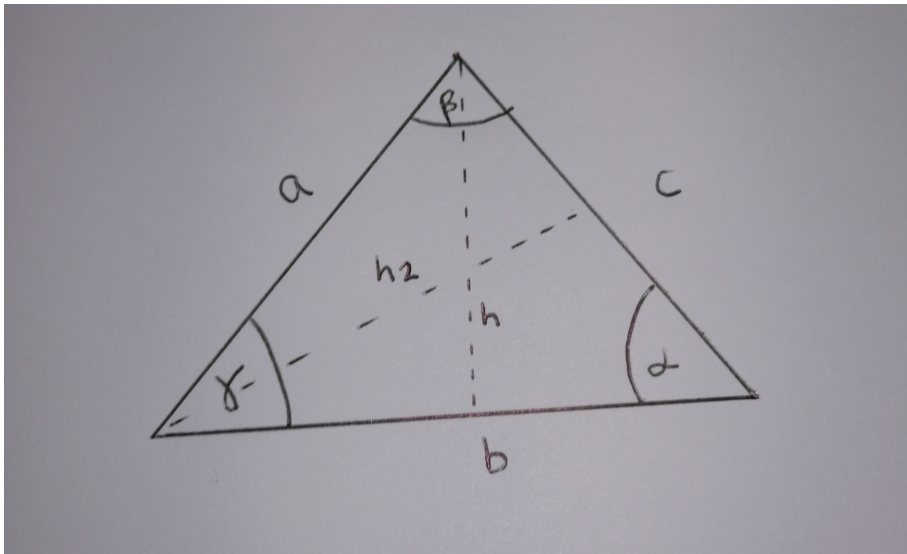
Suppose that we have a triangle with sides of length a , b and c with the angles opposite these sides labelled α , β and γ respectively.

Then

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}.$$

In an exam a student answered the question 'State and prove the Sine Rule' with the following:

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}.$$



Using basic Trigonometric formulas:

$$\sin\alpha = \frac{h}{c} \text{ so, } h = c\sin\alpha.$$

$\sin\gamma = \frac{h}{a}$ so, $h = a\sin\gamma$.
This means $c\sin\alpha = a\sin\gamma$.

Hence $\frac{\sin\alpha}{a} = \frac{\sin\gamma}{c}$.

Next we look at $\sin\beta = \frac{h}{b}$ therefore $h = b\sin\beta$.
Then $c\sin\alpha = b\sin\beta$ so $h = b\sin\beta$.

Hence $\frac{\sin\beta}{b} = \frac{\sin\alpha}{a}$. This means $a\sin\beta = b\sin\alpha$.

Therefore in conclusion

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}.$$

We have proved the initial equation given:

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}.$$

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