

Solitary Numbers, Their Density, and the Number Ten

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Solitary Numbers

Definition: Let n and m be any two positive integers. m and n is called a friendly pair, or a friend of n is m (or vice versa) if and only if $A(n) = A(m)$.

Defintion: N is called a solitary number (lonely number) iff n has no friend. In other words, there does NOT exist any integer $m \neq n$ such that $A(n) = A(m)$.

Solitary Numbers

- Given a natural number n . We define $\sigma(n)$ as the sum of all the divisors of n .
- $A(6) = 2$ and $A(28) = 2$ This is a friendly pair. So in order to find A of any number, you divide the divisors and divide it by that number.
- Proof: Assume n has a friend $m = n$. Since $(n, \sigma(n)) = 1$ then $A(n)$ is a reduced fraction. Also, m is a friend of n , then $A(m) = A(n)$. Hence m must be a multiple of n .
- Proof: Assume n has a friend $m = n$. Since $(n, \sigma(n)) = 1$ then $A(n)$ is a reduced fraction. Also, m is a friend of n , then $A(m) = A(n)$.
- Hence m must be a multiple of n .

How to Test Solitary Numbers

Step 1

The first step in testing solitary numbers is to list all of the divisors of the two numbers that you want to test.

Step 2

Once you've listed out the divisors, you add them all together, then divide by the number being tested.

Step 3

If the quotient of the two numbers is the same, then the numbers are friendly, if not, they're solitary.

Number Density

In 1996, Carl Pomerance told Dean Hickerson that he could prove that the solitary numbers have positive density, disproving a conjecture by Anderson and Hickerson (1977). However, this proof seems not to ever have been published (Hickerson 2002).

The numbers 1 through 9 have been identified as either solitary or friendly but the number 10 remains questionable because there hasn't been a friendly number found for it. What makes it such a unique number is that it also does not meet the qualifications for a solitary number. Because numbers can be tested up to infinity, it may take hundreds of years to find an equivalent solitary number or one may never be found and it will remain an unsolved math mystery.

The End